Example: Magnetization Currents

Problem:

Consider an **infinite cylinder** made of **magnetic** material. This cylinder is centered along the z-axis, has a **radius of 2 m**, and a **permeability** of $4\mu_0$.

Inside the cylinder there exists a **magnetic flux density**:

$$\mathbf{B}(\bar{r}) = \frac{8\mu_0}{\rho}\,\hat{\mathbf{a}}_\phi \qquad (\rho \le 1)$$

Determine the **magnetization current** $\mathbf{J}_{sm}(\bar{r}_s)$ flowing **on the surface** of this cylinder, as well as the magnetization current $\mathbf{J}_m(\bar{r})$ flowing **within the volume** of this cylinder.

Solution:

First, we note that we must know the **magnetization vector** $\mathbf{M}(\bar{r})$ in order to find the magnetization currents:

$$\mathbf{J}_m(\bar{r}) = \nabla \times \mathbf{M}(\bar{r}) \qquad \left[\frac{A}{m^2}\right]$$

$$\mathbf{J}_{sm}(\bar{r}_s) = \mathbf{M}(\bar{r}_s) \times \hat{a}_n \qquad \left[\frac{A}{m}\right]$$

But, we must know the **magnetic susceptibility** χ_m and the magnetic field $\mathbf{H}(\bar{r})$ to determine magnetization vector.

$$\mathbf{M}(\bar{r}) = \chi_m \, \mathbf{H}(\bar{r})$$

Likewise, we need to know the **relative permeability** μ_r to determine magnetic susceptibility:

$$\chi_m = \mu_r - 1$$

and we need to know the **magnetic flux density** $\mathbf{B}(\bar{r})$ to determine the magnetic field:

$$\mathbf{H}(\bar{r}) = \frac{\mathbf{B}(\bar{r})}{\mu}$$

But guess what! We **know** the relative permeability μ_r of the material, as well as the magnetic flux density within it!

$$\mu = 4\mu_0, \quad \therefore \mu_r = 4$$

$$\mathbf{B}(\bar{r}) = \frac{8\mu_0}{\rho}\,\hat{\mathbf{a}}_\phi \qquad (\rho \le 1)$$

Therefore, the **magnetic field** is:

$$\mathbf{H}(\bar{r}) = \frac{\mathbf{B}(\bar{r})}{\mu} = \frac{1}{4\mu_0}\frac{8\mu_0}{\rho}\,\hat{\mathbf{a}}_\phi = \frac{2}{\rho}\,\hat{\mathbf{a}}_\phi$$

and the **magnetic susceptibility** is:

$$\chi_m = \mu_r - 1 = 4 - 1 = 3$$

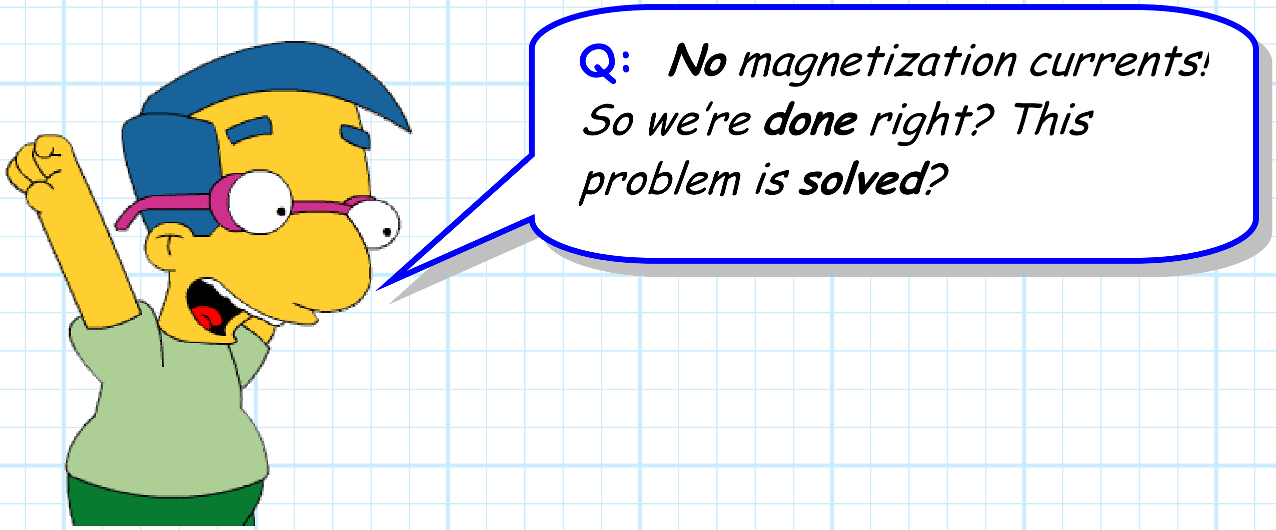
So the **magnetization vector** is:

$$\mathbf{M}(\bar{r}) = \chi_m \mathbf{H}(\bar{r}) = (3)\frac{2}{\rho}\hat{\mathbf{a}}_\phi = \frac{6}{\rho}\hat{\mathbf{a}}_\phi$$

Now (**finally!**) we can determine the **magnetization currents**:

$$\begin{aligned}\mathbf{J}_m(\bar{r}) &= \nabla \times \mathbf{M}(\bar{r}) \\ &= \nabla \times \left(\frac{6}{\rho}\hat{\mathbf{a}}_\phi\right) \\ &= 0\end{aligned}$$

The volume magnetization current density is **zero**—there is no magnetization current flowing **within** the cylinder!



A: Not hardly! Although there are no magnetization currents flowing **within** the cylinder, there might be magnetization currents flowing on the cylinder **surface** (i.e., $\mathbf{J}_{sm}(\bar{r}_s)$) !

$$\mathbf{J}_{sm}(\bar{r}_s) = \mathbf{M}(\bar{r}_s) \times \hat{\mathbf{a}}_n$$

Note for this problem, the unit vector normal to the surface of the cylinder is $\hat{\mathbf{a}}_n = \hat{\mathbf{a}}_\rho$.

Likewise, the magnetization vector **evaluated at the cylinder surface** (i.e., at $\rho = 2$) is:

$$\mathbf{M}(\bar{r}_s) = \mathbf{M}(\rho = 2) = \left.\frac{6}{\rho}\,\hat{\mathbf{a}}_\phi\right|_{\rho=2} = 3\,\hat{\mathbf{a}}_\phi$$

Therefore, the **magnetization current density** on the cylinder surface is:

$$\begin{aligned} \mathbf{J}_{sm}(\rho = 2) &= \mathbf{M}(\rho = 2) \times \hat{\mathbf{a}}_n \\ &= 3\,\hat{\mathbf{a}}_\phi \times \hat{\mathbf{a}}_\rho \\ &= -3\,\hat{\mathbf{a}}_z \qquad \left[A/m\right] \end{aligned}$$